PART 6: DOUBLE-STATION CAMERA WORK

1. Computation of the direction of the optical axis

Different methods exist for computing where to direct a camera for double-station photography. The following section describes one possible way to get some results.
We assume meteors produce their maximum luminosity at about 90 km above the Earth's surface. This means we have to observe the same area at 90 km height in the atmosphere from two or more geographic locations. Mostly we take a fixed point at 90 km altitude above a set location on the Earth which is at about an equal distance away from both observing stations. (e.g. at about 50 km distance). This point, called the projection point, can be determined with the help of an atlas or with a topographic map. First of all the geographic coordinates of this projection point (φ_p, λ_p) have to be determined.
For each observing station we determine the height h and the azimuth Az at which the camera has to be aimed, to be sure to photograph the area at 90 km above this projection point. A few formulae using trigonometry allow us to solve this problem. An example will clarify the working procedure:
Step 1: Calculation of the angular distance from observing station to projection point. Since the computations are analogous for each observing station we need work with only one station here.

$$X = \arccos(\sin\varphi_p \cdot \sin\varphi_o + \cos\varphi_p \cdot \cos\varphi_o \cdot \cos(\lambda_p - \lambda_o))$$

$$Y = 60 \cdot 1852\text{m} \cdot X$$

X is the angular distance on the spherical Earth's surface between the two places (in °). As one arc minute corresponds to 1852 meters we find Y in meters.
Step 2: Computation of the elevation angle of the camera. Once we know the distance Y, it is very easy to determine the elevation h at which we have to aim the camera.
The sketch below makes it clear that :

$$\tan h = \frac{90\text{km}}{Y}, h = \arctan\frac{90\text{km}}{Y}$$

Step 3: Computation of the azimuth Az of the camera direction. The azimuth is found as follows:

$$\sin(270° - Az) = \frac{B''}{Y} = \frac{60 \cdot 1852 \cdot (\varphi_p - \varphi_o)}{Y}$$

$$\cos(270° - Az) = \frac{A''}{Y} = \frac{\cos(\varphi_p - \varphi_o)/2 \cdot 60 \cdot 1852 \cdot (L_p - L_o)}{Y}$$

where $\tan(270° - Az) = B''/A''$ and thus $Az = 270° - \arctan(B''/A'')$
The values of $\sin(270° - Az)$ and $\cos(270° - Az)$ allow the determination of the right quadrant where the azimuth is to be found. We remind readers who left school some time ago of the following rules:
$\sin +$ and $\cos +$ then $0° \leq (270° - Az) \leq 90°$
$\sin +$ and $\cos -$ then $90° \leq (270° - Az) \leq 180°$
$\sin -$ and $\cos -$ then $180° \leq (270° - Az) \leq 270°$
$\sin -$ and $\cos +$ then $270° \leq (270° - Az) \leq 360°$
or use the following circle:

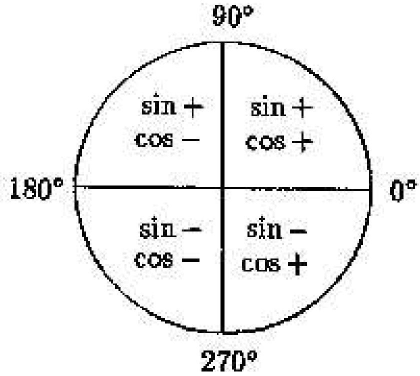

Figure 6-1: Sign of the sin and cos functions.

Some pocket calculators allow the conversion of an angle into the right quadrant automatically.
Step 4: Computation of the declination and right ascension.
To point the camera exactly it is recommended to convert the h and Az to right Ascension α and declination δ, in order to find the selected area to be photographed in a star atlas.
The declination we compute from:

$$\delta = \arcsin(\sin\varphi_o \cdot \sin h - \cos\varphi_o \cdot \cos h \cdot \cos Az)$$

Once we know δ, we can compute the hour angle $\tau = (\theta - \alpha)$:

$$\sin\tau = \frac{\cos h \cdot \sin Az}{\cos\delta}$$

$$\cos\tau = \frac{\cos Az \cdot \cos h + \cos\varphi_o \cdot \sin\delta}{\sin\varphi_o \cdot \cos\delta}$$

Analogous to the computation of the azimuth, the values of $\sin(\theta - \alpha)$ and $\cos(\theta - \alpha)$ allow us to determine the right quadrant for $\tau = (\theta - \alpha)$. Once we have obtained $\tau = (\theta - \alpha)$ we can derive the right ascension of the camera direction when we know the local sidereal time θ. The local sidereal time can be obtained from any good astronomical yearbook or by a simple computational procedure (see for instance "Norton's Star Atlas"; Tirion, 1981).

Worked example:
The Perseids were observed on 12 August 1981 from two stations, one being Brustem in Belgium ($\varphi_1 = 50°48'29''$, $\lambda_1 = 5°13'56''$) and the other being Mechelen also in Belgium ($\varphi_2 = 51°00'59''$, $\lambda_2 = 4°29'59''$). The mean time of the photographic project was planned to be 1^{h} UTC. Since the Perseid radiant is in an easterly direction at the mean time of the photography, the projection point was chosen such that both observers photograph an area of the sky in a southerly direction. We chose a projection point at $\varphi_p = 50°35'$ and $\lambda_p = 4°45'$.
Step 1:
$X = \arccos(\sin 50.808 \cdot \sin 50.583 + \cos 50.808 \cdot \cos 50.583 \cdot \cos(5.2322 - 4.75))$
$Y = 60 \cdot 1852\mathrm{m} \cdot 0.378 = 42031\mathrm{m}$

Step 2:
$\tan h = \frac{90000}{42031}$
$h = \arctan 2.14 = 65°$
Step 3:
$A'' = -33782$m
$B'' = -24974$m
and $\tan(270° - Az) = 0.73925$
$\sin(270° - Az) = -0.59445$
$\cos(270° - Az) = -0.80413$
and since both sin and cos are negative then:
$180° \leq (270° - Az) \leq 270°$,
e.g. $(270°\text{-}Az) = 216.4737°$
so $Az = 270° - 216.4737° = 53°32'$
Step 4:
$\delta = \arcsin(\sin 50°48' \sin 64°59' - \cos 50°48' \cos 64°59' \cos 53°32')$
$\delta = \arcsin(0.54332)$
$\delta = 32°55'$
Furthermore
$\sin(\theta - \alpha) = 0.405213$
$\cos(\theta - \alpha) = 0.914409$
and hence both sin and cos are positive values, so we get
$0° \leq (\theta - \alpha) \leq 90°$and thus
$(\theta - \alpha) = 23°54'$
For the local sidereal time we found $22^{\mathrm{h}}42^{\mathrm{m}}22^{\mathrm{s}}$ or $\theta = 340.5939°$so $\alpha = -23°54' + 340°36' = 316°42'$
Summarizing the result:
Observing station 1 Brustem
$h = 65°$, $Az = 54°$
$\alpha = 317°$, $\delta = 33°$
As an exercise, compute the camera position for the observer in Mechelen. You should find the following results: $h = 60.4°$, $Az = 340°$, $\delta = +22.6°$and $\alpha = 351°$.
If you have a country-wide network with many observing stations which each have several cameras we strongly recommend you to program the organization of a multiple station project on a PC. Always check your starting data as an error in the setting up of the camera directions will result in disappointed photographers when they find out their photographed meteors were not simultaneous, despite all the careful planning!

2. Intersection of camera fields at a given height and determination of the camera directions

To determine the direction at which a camera has to be aimed for simultaneous photography, most people consider only the intersection of the optical axes at a height h, typically 90 km for most meteor photography.
This method does not guarantee the cameras will not be pointed close to the horizon. The following numerical-graphical method prevents this.
We assume that cameras with stereographic projection of the image (such as reflex cameras), with a focal length f (mm) and rectangular negatives a(mm) $\times b$ (mm) are used.

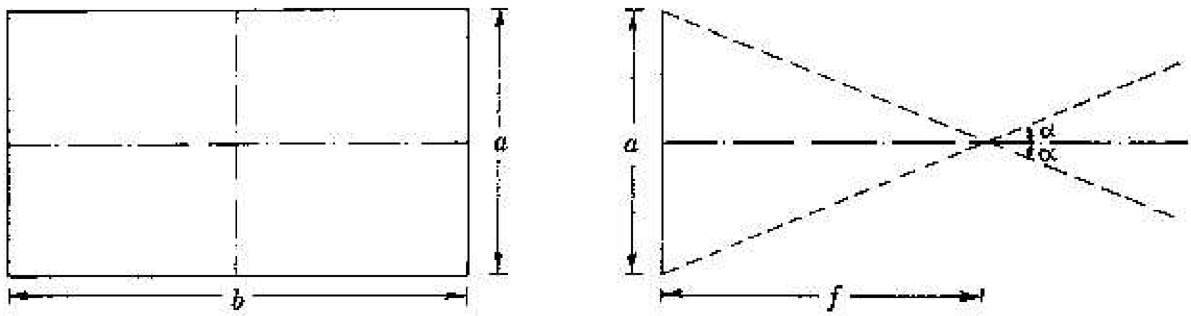

Figure 6-2: Field of view of a camera equipped with a lens of focal length f (mm). The format of the negative is a (mm)$\times b$ (mm). This gives a field in the sky of $2\alpha \times 2\beta$ degrees.

From Fig. 6-2 (right) we see, that the semi image angle α (vertical) is given by:

$$\tan\alpha = \frac{a}{2f}$$

The semi image angle β (horizontal) is then given by:

$$\tan\beta = \frac{b}{2f}$$

Example: consider a small-frame camera with a standard lens: $a = 24$ mm, $b = 36$ mm, $f = 50$ mm then: $\tan\alpha = 0.24$ and $\tan\beta = 0.36$, or $\alpha = 13.5°$ and $\beta = 19.8°$, respectively. The camera will then image a field of $2\alpha \times 2\beta$, which is 27.0° by 39.6° in this example.
The optical axis is pointed at an elevation η (°), whereby the edge b is parallel to the horizon. We work relative to a flat Earth, which makes our results less accurate for $\eta - \alpha < 25°$ (to be avoided in practice anyway because of light pollution or haze close to the horizon, the excessive distance to the meteor, etc.)
In Fig. 6-3 the field of the camera is shaded. The camera is located at point W at the Earth's surface. The surface at the height level h photographed in the atmosphere can be reconstructed when the dimensions b_1, b_2, d_1 and d_2 are known, therefore we need to compute these.
The formulae are:

$$p = -h \cdot \tan\eta \tag{1}$$

$$p \cdot d_o = -h^2 \tag{2}$$

$$d_o = \frac{h}{\tan\eta} \tag{3}$$

$$d_1 = \frac{h}{\tan(\eta + \alpha)} \tag{4}$$

$$d_2 = \frac{h}{\tan(\eta - \alpha)} \tag{5}$$

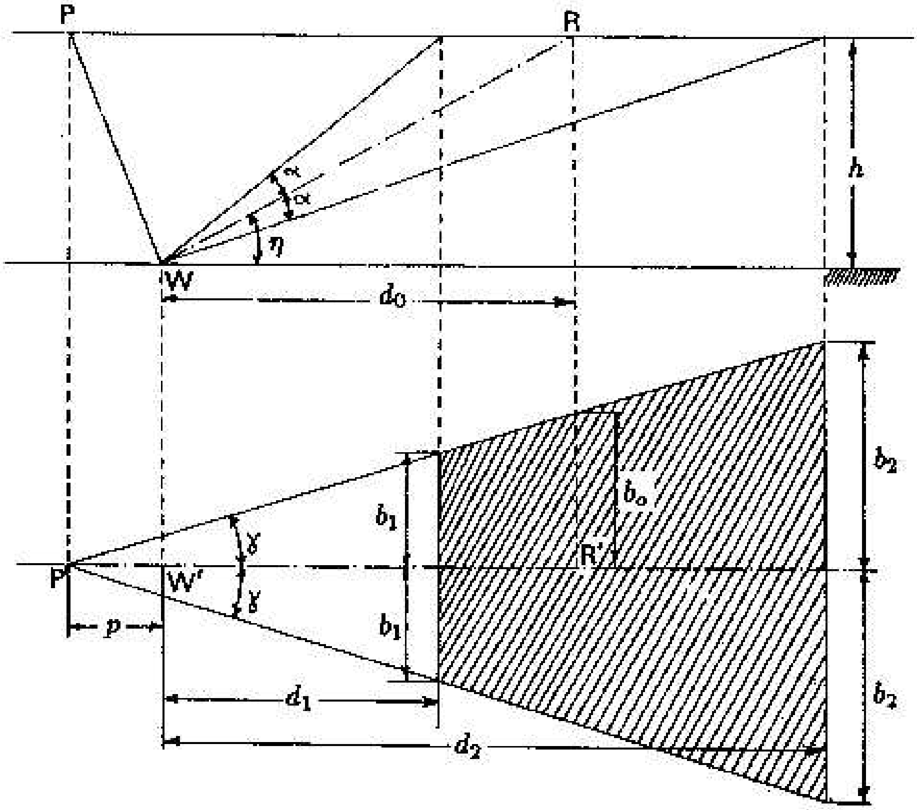

Figure 6-3: Camera field in the atmosphere at height level h. **R** denotes the camera field center in the sky. The lower part shows the situation projected onto the Earth's surface.

The computation of b_1 and b_2 is more complicated. First compute:

$$b_o = \frac{\tan\beta \cdot h}{\sin\eta} \tag{6}$$

Next:

$$\tan\gamma = \frac{b_o}{d_o - p} = \frac{\tan\beta}{\sin\eta}\frac{1}{\frac{1}{\tan\eta} + \tan\eta} = \tan\beta \cdot \cos\eta \tag{7}$$

$$b_1 = (d_1 - p)\cdot\tan\gamma = h\cdot\tan\gamma\left(\frac{1}{\tan(\eta+\alpha)} + \tan\eta\right) \tag{8}$$

$$b_2 = (d_2 - p)\cdot\tan\gamma = h\cdot\tan\gamma\left(\frac{1}{\tan(\eta-\alpha)} + \tan\eta\right) \tag{9}$$

$$b_1 = b_0(d_1 - p)/(d_0 - p) = b_0\left(1 + \frac{(d_1 - d_0)}{(d_0 - p)}\right) \tag{8a}$$

$$b_2 = b_0\left(1 + \frac{(d_2 - d_0)}{(d_0 - p)}\right) \tag{9a}$$

The formulae (8a) and (9a) are better than (8) and (9) for numerical calculations when η is close to 90° (p very large). It is also interesting to know the surface area S (in km^2) of the camera field:

$$S = (d_2 - d_1)\cdot(b_1 + b_2) \tag{10}$$

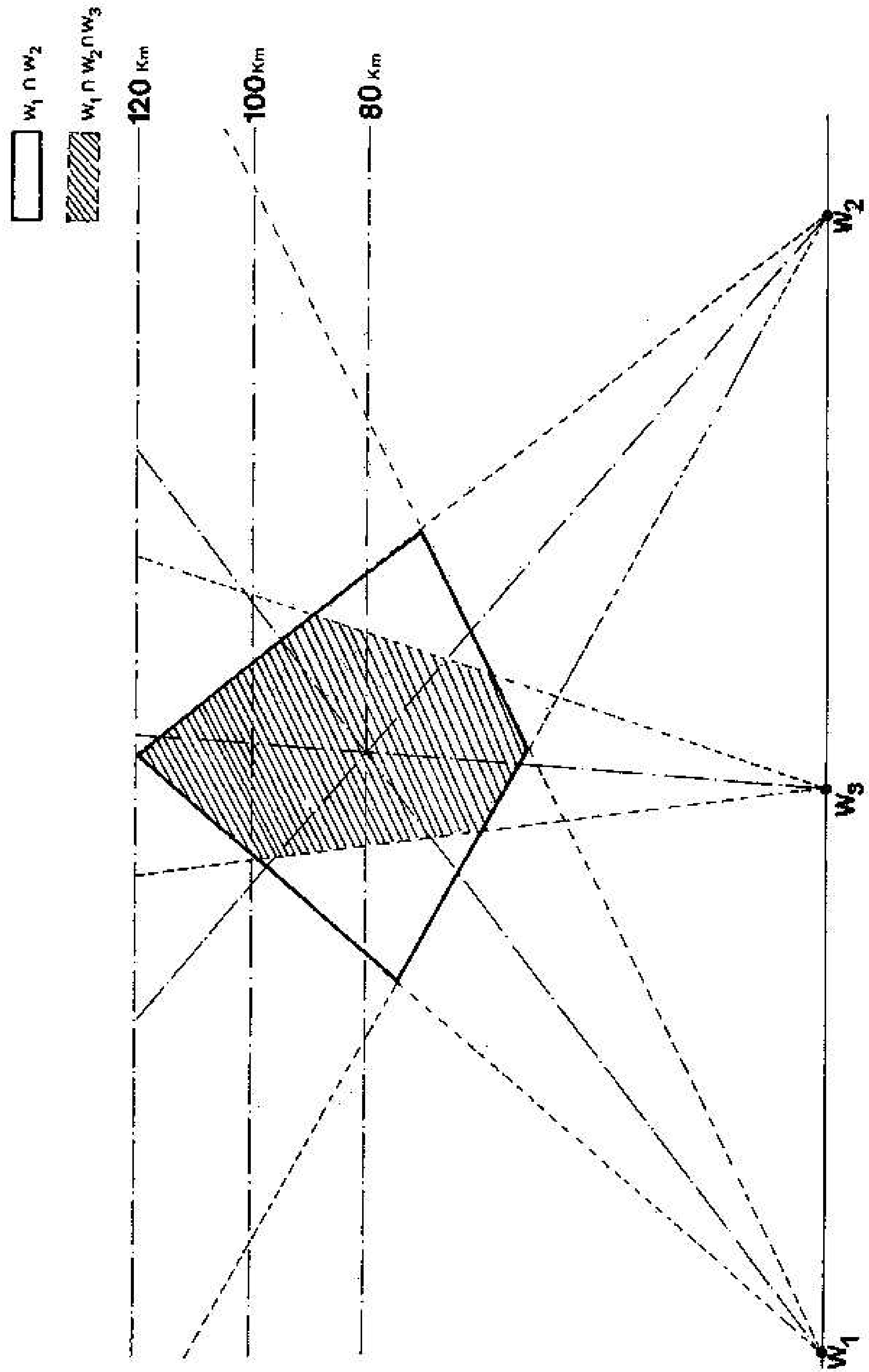

Figure 6-4: Intersection of camera fields in the meteor level between 80 and 120 km shown as a vertical cut. (from Vanmunster, 1986, p. 37)

An example may clarify this method: the camera from the example above is directed at an elevation η of 60°. We compute the results for a height $h = 90$km and $h = 70$km, respectively.

		$h = 90$km	$h = 70$km
(2)	p	−155.9 km	−121.2 km
(3)	d_o	52.0	40.4
(4)	d_1	26.7	20.7
(5)	d_2	85.4	66.4
(6)	b_o	37.4	29.1
(7)	$\tan\gamma$	0.18	
(8)	b_1	32.9	25.6
(9)	b_2	43.4	33.8
(10)	S	4473 km^2	2715 km^2

3. Application

The aim is to optimise the intersection of the camera fields (at the mean height of bright meteors) for photography from two or more observing stations. Since both azimuth and elevation must be determined, it is very useful to draw the horizontal projection of the camera fields on a transparent sheet for a few heights h.
This method allows us to verify which cameras have to be mobilized at the different stations in order to cover a well-defined part of the sky. Remember however that some other factors will still play a role in the determination of the positioning of cameras, such as differences in the light sensitivity of the cameras. Therefore coordination of multiple station projects needs to be centralized by local workers.

4. Computation of the region of sky photographed

When a multiple station project is prepared only one point is given to direct the camera; the point at which the optical axis must be aimed (R). Where several cameras are used at one place, it is interesting to know whether some camera fields are overlapping and if so by how much. Therefore the corners of each camera field are computed as projected onto the celestial sphere. In such a spherical projection, the camera field is of course no longer a rectangle (Fig. 6-5).

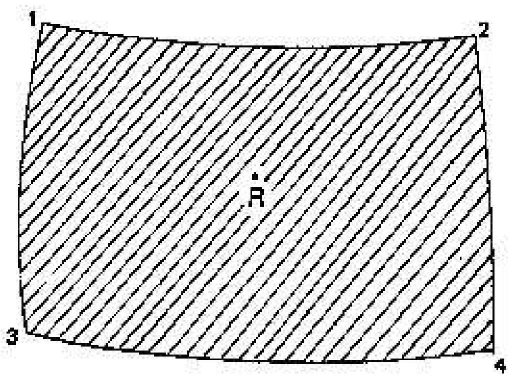

Figure 6-5: Shape of a camera field projected onto the celestial sphere.

The required calculations remain simple as shown in the following method. When we determine the efficiency E of a camera for meteor photography we find that the size of the camera field is given by:

$$A = 2 \cdot \arctan \frac{a}{2f} \cdot 2 \cdot \arctan \frac{b}{2f}$$

For a camera with $f = 50$ mm and film dimensions 24 mm×36 mm ($a \times b$) the camera field A measures 39.6° ×27°. Assuming that the camera is mounted horizontally, we can obtain the upper limit of the camera field (this is the elevation of points 1 and 2 in the figure above) by adding 27°/2 or 13.5° to the elevation of the direction point R. Analogously we can determine the lower limit by reducing the elevation of R by 13.5° (= elevation of points 3 and 4).
Example: when the elevation of the direction point R is 60°, then the elevation of points 1 and 2 (upper limit) is 73.5° and of the points 3 and 4 (lower limit) 46.5°.
Now we have still to determine the azimuth of the corners of the camera field. From Figure 6-6 we see that the angle on the azimuth scale becomes larger as we approach the horizon, in other words, the angle on the azimuth scale increases by a factor $\sec h = (\cos h)^{-1}$. The value that has to be added to (or subtracted from) the azimuth value of R equals $\sec h \cdot (39.5°/2)$.

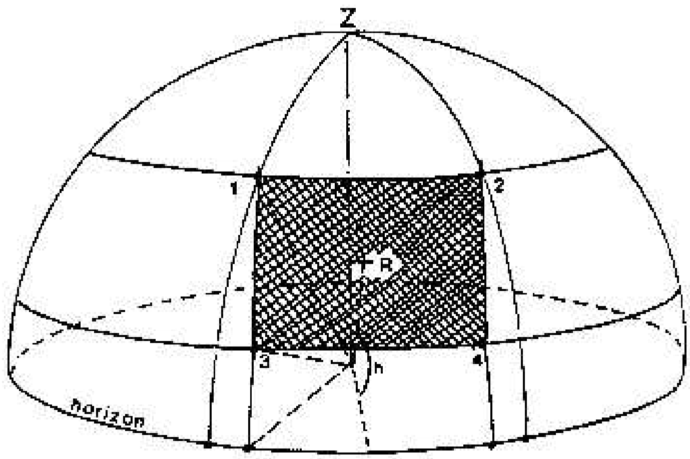

Figure 6-6: Camera field on the celestial sphere. The indicated points are explained in the text.

Example: For the points 1 and 2 we found an elevation of 73.5°. The angle on the azimuth scale between the 'central meridian' and the upper corners of the camera field are found to be $(39.6°/2)\cdot \sec 73.5° = 69.7°$. Assuming that the azimuth value of the direction point R was 200°, we find an azimuth for corner 1 of the camera from 200°−69.7°=130.3° and for corner 2 from 200°+69.7°= 269.7°.
Exercise: Compute the azimuths of the other corners.
Results:
Corner 3: $h = 46.5°$; $a = 171.2°$
Corner 4: $h = 46.5°$; $a = 228.8°$
Of course when several cameras are used at one observing station, it is useful to arrange to minimize the overlapping regions of camera fields on the sky via the above method!

5. Optimising the camera fields

As the basic principles of double station photography have now been explained we can find out whether or not every meteor within the camera field will be caught at the two stations.
Let us look once again at Fig. 6-3. The bottom of our camera field photographs a region of the sky which is much closer to the horizon than the other parts of the image. Meteors that appear in the bottom region of the photo occur at a much larger distance away from the observer than the meteors caught in the center. Due to this greater distance these meteors appear fainter and the probability of photographing them decreases considerably. The loss in luminosity due to the distance relative to a

standard distance of 100 km is given by:

$$\Delta m = 2.5 \cdot \log\left(A^2/100^2\right)$$

where A is the distance to the meteor in km.
The composition of the gases in the atmosphere through which we observe will also reduce the meteor's luminosity, due to absorption. The absorption can be approximated by the following equation (valid for a pure terrestrial atmosphere):

$$\Delta m' = \frac{0.38}{\sin(\eta - 1)} = 0.38 \cdot \csc(\eta - 1)$$

Where η is the elevation for which $\Delta m'$ is to be determined.
For a polluted atmosphere $\Delta m'$ will be greater as the 'constant' value of 0.38 will no longer be valid. In populated areas, the ideal values of 0.38 will never occur, but will be at least 0.6 to 1.0! The absorption value varies strongly from one region to another and from one night to the next. The total loss in luminosity Δm for a photographed meteor thus becomes:

$$\Delta M = \Delta m + \Delta m'$$

. The real loss in hourly rates is also determined by the mass distribution of the meteoroids, described by the so-called population index r. For sporadic meteors, r varies mostly around ≈ 3.0. For shower meteors, r is generally smaller: $2.0 \leq r \leq 3.0$. The factor which reduces the number of meteors recorded C due to the loss of luminosity is written as: $C = r^{-\Delta M}$. It is obvious that the lower the elevation of the camera, the fewer meteors we will photograph due to the loss in luminosity. There is, however, also a gain. The lower we aim the camera towards the horizon, the larger the intersection volume with the meteor layer in the atmosphere becomes. The photographed volume at 100 km height V increases by the following factor as a function of the camera elevation η:

$$V = \frac{1}{\sin \eta^3} = \csc \eta^3$$

In order to find out whether or not we gain anything by selecting a given elevation of the camera, we compute T:

$$T = \csc^3 \eta \times r^{-(0.38 \csc \eta - 1) - 5 \log \sin \eta)}$$

$T = 1$ for a camera centered at the zenith.
$T < 1$ means that fewer meteors will be photographed than overhead.
$T > 1$ means that more meteors will be photographed than in the zenith.
For a perfectly transparent sky (absorption coefficient = 0.38) it turns out we gain in terms of photographic hourly rates for most shower meteors ($r < 3.0$) by aiming the camera at a less than zenithal elevation (e.g. 30°). For meteor streams which are richer in faint meteors it turns out the optimal elevation is around 50°. In the case of shower meteors we also have to consider systematic differences in angular velocity dependent on the direction and distance to the radiant (cf. section 8 in Part 1.)
In most cases however one will work where the sky is far from perfectly transparent, as most observers do their observational work close to their homes, often near towns or industrial areas. Because of the large absorption coefficients it is not recommended to aim a camera at a low elevation in these cases. At most observing stations the photographic hourly rate will decrease as the zenith distance of the camera field increases. This explains why photographers can have very different degrees of success with the same kind of optics and film, but at different sites.

In double station photography it is important to try to photograph an area in such a way that the absolute limiting magnitude for meteors in both camera fields is, as far as possible, identical. Remember, however, that:

- The camera fields must cover about the same volume in the atmosphere.
- The distance to the other stations of a network must be about equal.

At this stage it becomes clear that the optimization of a camera network becomes a complex organizational job!
Even when the above criteria are fulfilled there is still one important element to be considered. There is no guarantee that the photographed double station meteors will yield good results, as for instance, the trail of the meteor and the two observing places may lie in the same plane. The angle of convergence of the two photographed meteors will then be zero. Such a case is a most annoying problem, giving very poor computational results. In view of this problem we need to take the geometric conditions of occurrence for shower meteors into account. The best way to improve our camera positions is to examine the entry angles of the shower meteors we intend to try to photograph by simulating the actual circumstances.

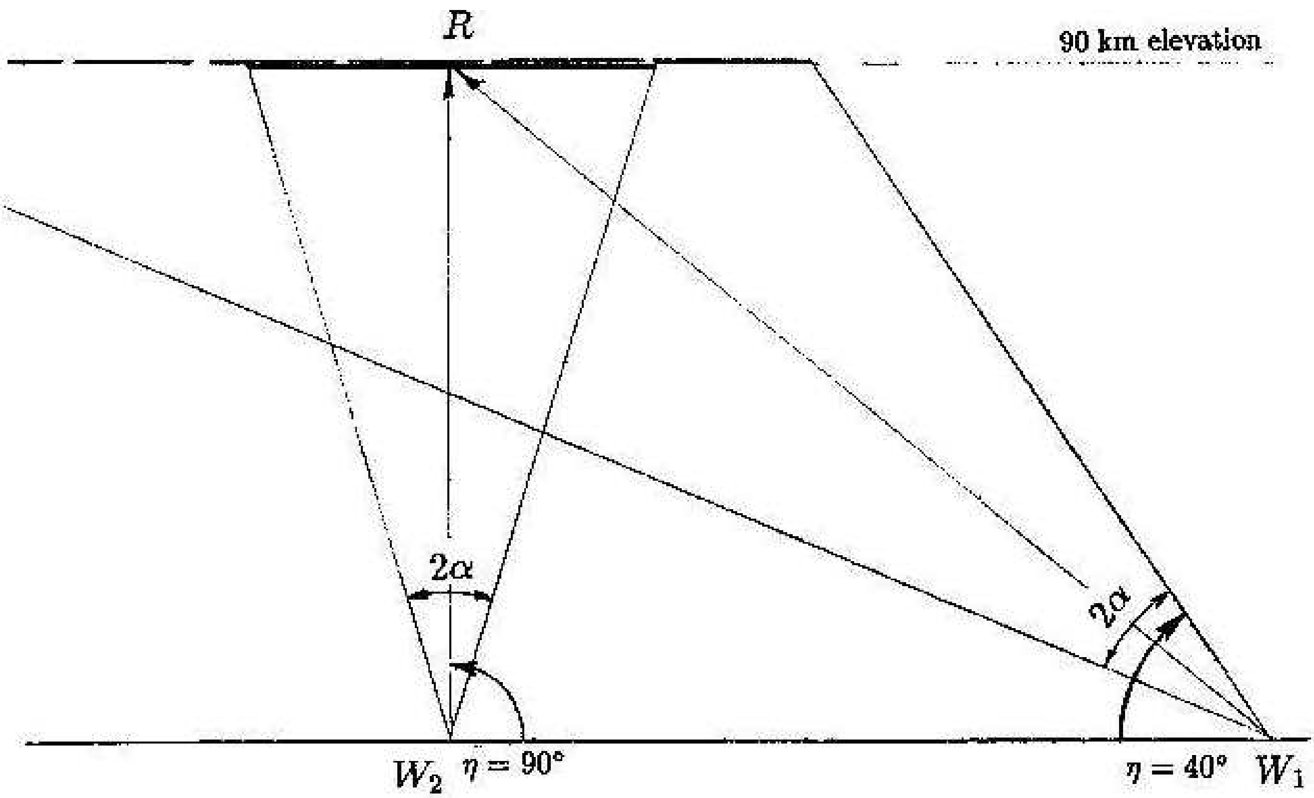

Figure 6-7: Two photographers aim their cameras, equipped with identical lenses, at the same point R in the atmosphere. Observer W_1 aims at an elevation of 40°, W_2 at 90°. In this case we may question the usefulness of a double station campaign: in the example, the camera at W_2 photographs only a very small part of the camera field of W_1 (dashed).

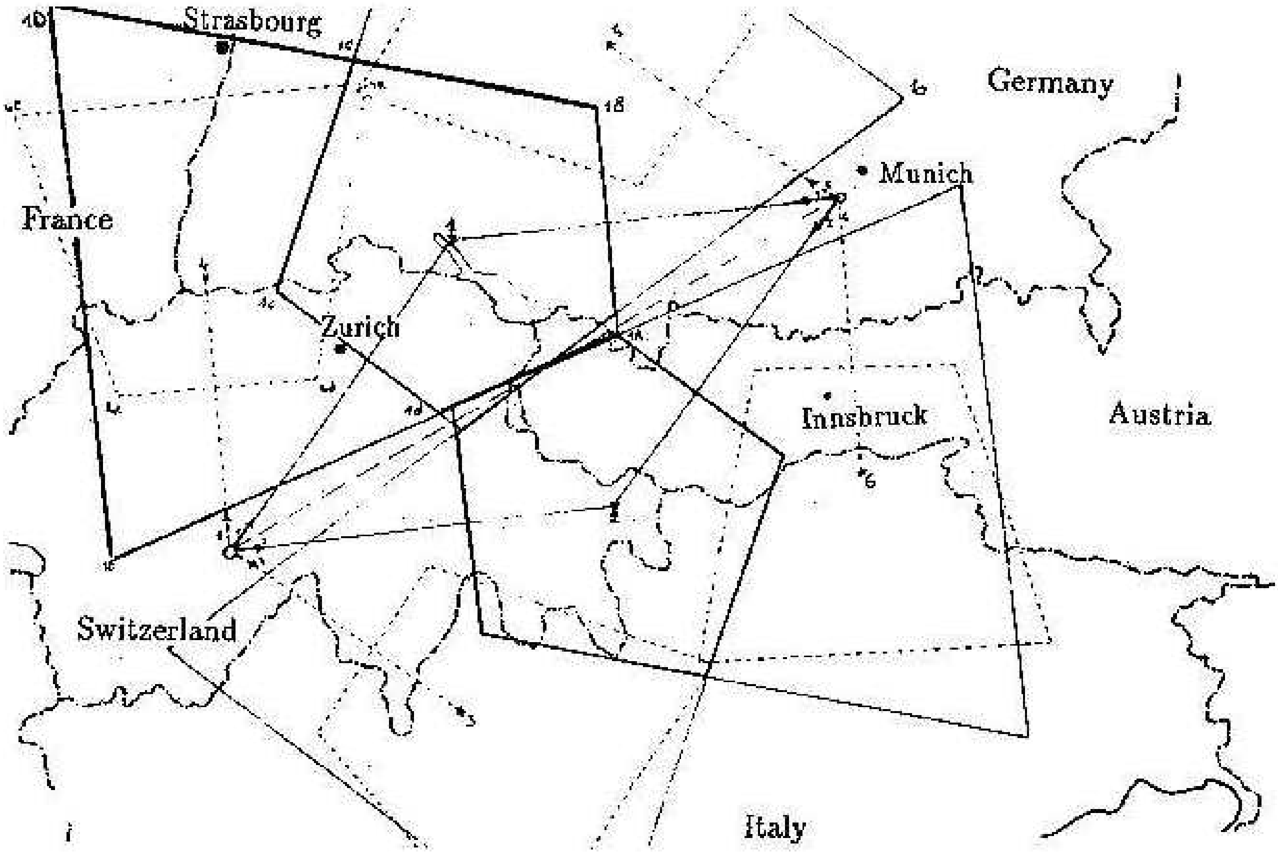

Figure 6-8: The camera fields for two stations, one in Switzerland, the other near Munich in Germany, as used during an actual observing campaign as projected onto a map. As can be seen, the camera fields cover large areas above France, Switzerland, Austria, Italy and Germany. Corners of three fields are marked (1A–E, 1a–d, and 4a–d), which correspond to the positions indicated in Fig. 6-9.

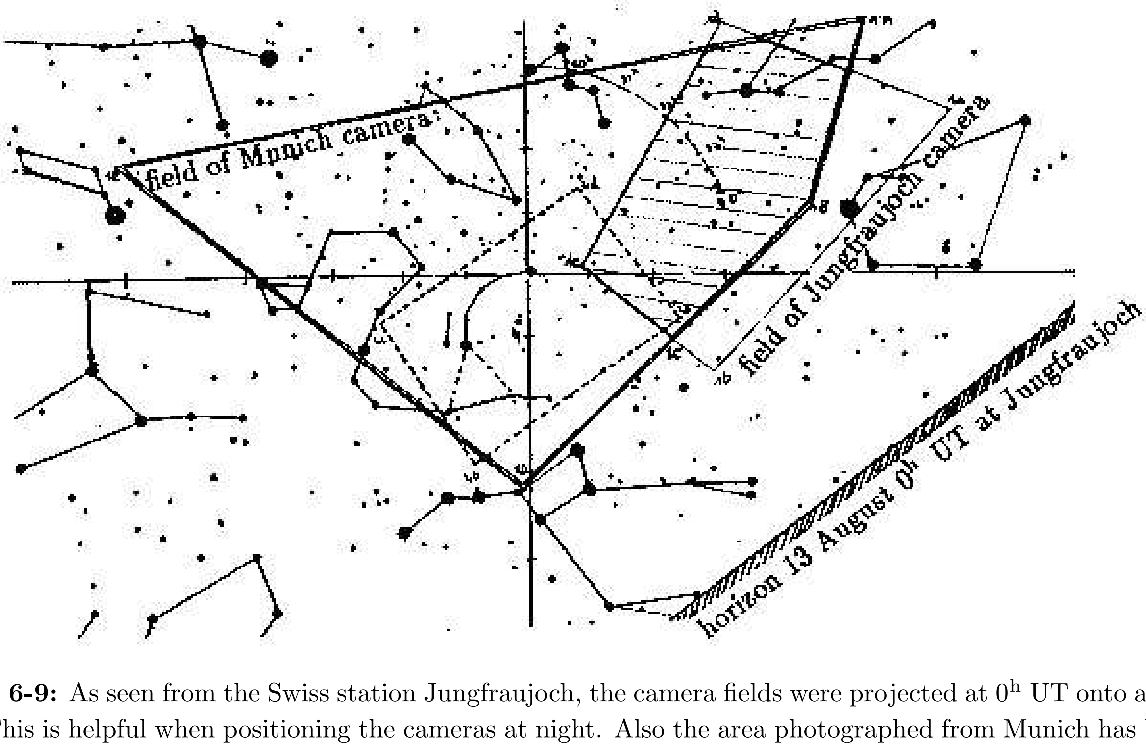

Figure 6-9: As seen from the Swiss station Jungfraujoch, the camera fields were projected at 0^{h} UT onto a star map. This is helpful when positioning the cameras at night. Also the area photographed from Munich has been indicated (dashed field), which shows only a small part of this camera field is common with the corresponding one at Jungfraujoch.

6. Simulating meteor trails for tests

We will not generate meteors at random, but will instead select where we will let the meteor appear (1), from which radiant it radiates (2), the time (3) and its absolute magnitude (4). With these four starting elements we can begin the simulation. The procedure, step by step is as follows:
– We compute the azimuth a and elevation h_{Rad} of the radiant at the selected time and for the geographical position φ_b, λ_b above where the meteor starts at height h_b.
– We compute the position of the ending point h_e of the meteor. From the length l(km), the azimuth and the elevation of the radiant: $h_e = h_b - l \times \sin h_{\mathrm{Rad}}$

$$\lambda_e = \lambda_b \pm \frac{l \cos h_{\mathrm{Rad}} \sin a_{\mathrm{Rad}}}{1852 \cdot \cos \varphi_e \cdot 60}$$
$$\varphi_e = \varphi_b \pm \frac{l \cos h_{\mathrm{Rad}} \cos a_{\mathrm{Rad}}}{1852 \cdot 60}$$

note: for some showers the mean values for l and h_b can be found in research papers.
– Calculate from $h_e, \varphi_e, \lambda_e$ and $h_b, \varphi_b, \lambda_b$ the azimuth and elevation for each observer W_1 and W_2 at locations φ_1, λ_1 and φ_2, λ_2.
– Compute: (W_{ix} are vectors)

$$W_{1b} = (\cos a_{b1} \cos h_{b1}, \sin a_{b1} \cos h_{b1}, \sin h_{b1})$$
$$W_{1e} = (\cos a_{e1} \cos h_{e1}, \sin a_{e1} \cos h_{e1}, \sin h_{e1})$$
$$W_{2b} = (\cos a_{b2} \cos h_{b2}, \sin a_{b2} \cos h_{b2}, \sin h_{b2})$$
$$W_{2e} = (\cos a_{e2} \cos h_{e2}, \sin a_{e2} \cos h_{e2}, \sin h_{e2})$$

– Compute the angular length $L(°)$ of the meteor seen from both observing stations.
At W_1: $L_1 = \arccos(W_{1b} W_{1e})$ (vectorial product)
at W_2: $L_2 = \arccos(W_{2b} W_{2e})$
– Compute the apparent magnitude M_a seen from W_1 and W_2: derive the mean distance from observer to meteor A and the elevation at which the meteor appear for that observer. Next compute the magnitude correction due to the distance, Δm, and the loss in luminosity due to absorption $\Delta m'$. The apparent magnitude M_a then becomes:

$$M_a = M_{abs} + \Delta m + \Delta m'.$$

– Calculate the angle of convergence σ:

$$u_1 = W_{1b} \times W_{1e}$$
$$u_2 = W_{2b} \times W_{2e}$$
$$\sigma = \arccos \frac{u_1 \cdot u_2}{|u_1| - |u_2|}$$

As a final visual presentation we can project these meteors on to a map. We translate (a, h) of the meteor's position to (α, δ) and project (α, δ) as (x, y) coordinates onto a gnomonic map. This will give a much better insight into all the elements, such as angular length $L(°)$ of the meteor, its angular distance from the radiant, the angle of convergence between the meteor trails as seen from different stations, etc. With such a method it would even be possible to simulate an entire observing campaign. In the case where conditions for a double station project need to be respected, it is recommended to direct the camera at about 30° from the radiant as at such a distance from the radiant the angular velocity and trail lengths of the stream are most favourable.

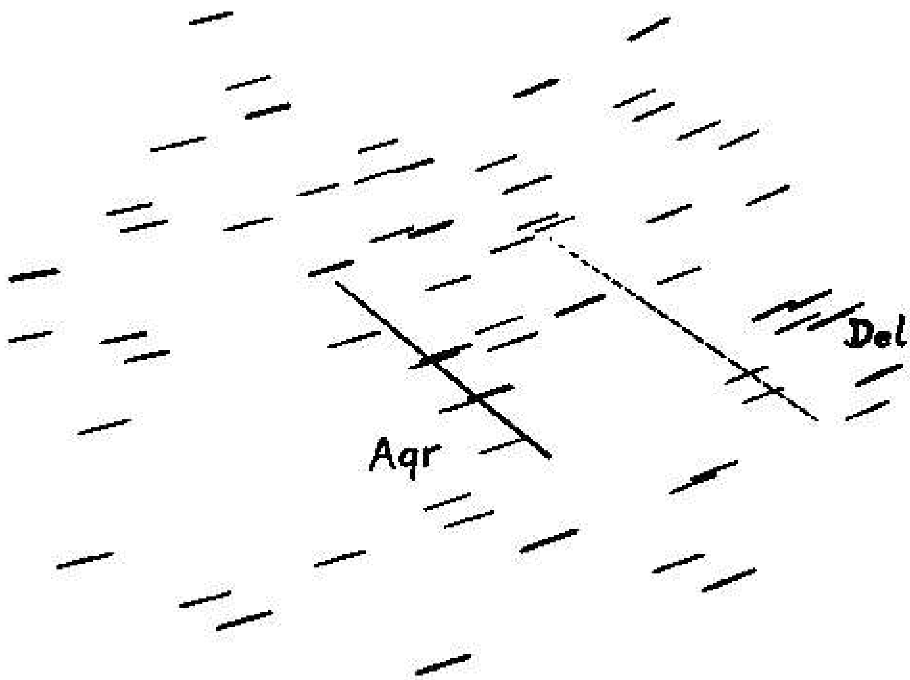

Figure 6-10: To get an impression about the parallax of meteor trails photographed from two stations separated by 25 km, we show an example of a Perseid meteor of -2^{m}. The exposures were taken on 1975 August 13 between $22^{\mathrm{h}}40^{\mathrm{m}}$ and $22^{\mathrm{h}}50^{\mathrm{m}}$ local standard time. The meteor appeared at $22^{\mathrm{h}}44^{\mathrm{m}}$. We superposed both meteor trails into one frame to show the parallax. The trail photographed by the easterly station appears west of the other trail. It is interrupted by a rotating shutter with 25 breaks per second.

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